# THE TEMPERATURE FIELD IN INDUSTRIAL CLOTH USED IN A THREE-LAYER SYSTEM

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The problem of determining the temperature field in the cloth of the three-layer system in the paper contact-drying process is analyzed.

It is interesting, both practically and theoretically, to consider the problem of determining temperature and mass distribution in a three-layer system which comprises two sheets of heated drying cloth with moist paper in between [1].

This problem can, with certain assumptions concerning the conditions under which the cloth sheets and the surrounding medium interact, be reduced to the analysis of a two-layer system: moist paper and heated cloth. The cloth sheets on both sides of the paper are considered to have the same structural-mechanical and physicochemical properties. i.e., the system is considered to be symmetrical with respect to the center plane of the paper.

In a rigorous statement of the problem, it is required to solve two coupled systems of differential equations for the combined heat and mass transfer [2]:

$$c_k\gamma_{0k}\frac{\partial t_k}{\partial \tau} = \mathrm{div}(\lambda_k\nabla t_k) + \varepsilon_k\rho_k\gamma_{0k}c_{mk}\frac{\partial \theta_k}{\partial \tau}, \tag{1}$$

$$c_{mk}\gamma_{0k}\frac{\partial \theta_k}{\partial \tau} = \mathrm{div}\left[\lambda_{mk}\nabla\theta_k + \lambda_{mk}\delta_k\nabla t_k\right], \tag{2}$$

where k = 1, 2.

The solution to the system of equations (1), (2) based on the uniqueness conditions for the one-dimensional problem with the initial conditions

$$t(x,\ 0) = f_1(x), \quad \theta(x,\ 0) = f_2(x), \tag{3}$$

as well as on the fourth-kind boundary conditions at the cloth–paper contact and the third-kind boundary conditions at the outer cloth surface is rather difficult [2].

The problem as stated becomes much simpler, however, when the form of functions $\varphi(\tau)$ and $\psi(\tau)$ expressing the equality of heat and mass flows across the cloth–paper boundary is known:

$$\lambda_1\frac{\partial t_1}{\partial x} + (1-\varepsilon)\rho_1 j_{m1}(\tau) = \lambda_2\frac{\partial t_2}{\partial x} = \varphi(\tau), \tag{4}$$

$$j_{m1}(\tau) = j_{m2}(\tau) = \psi(\tau). \tag{4'}$$

In this case the contact problem (1), (2) splits into two independent constraint problems for the paper and for the cloth.

Let us consider the statement and the solution of the constraint problem for the cloth.

It was shown in [1], on the basis of performed experiments, that a substantial amount of moisture from the paper dried by contact with cloth sheets – wool and cotton are used most often in practice – is

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extracted as vapor which does not condense in the hotter cloth. The mass of vapor is negligibly small here in comparison with the mass of cloth (in practice, the moisture content in the cloth varies from 2 to 3% throughout the drying process). In other words, the effect of vapor passing through the cloth may be disregarded in any consideration of the temperature field in the cloth. If that is so, then the problem of determining the temperature field and the moisture field in the cloth reduces to solving the system of equations:

$$\frac{\partial t_2}{\partial \tau} = a_2 \frac{\partial^2 t_2}{\partial x^2},$$
$$\frac{\partial u_2}{\partial \tau} = a_m \frac{\partial^2 u_2}{\partial x^2} + a_m \delta \frac{\partial^2 t_2}{\partial x^2}. \qquad (5)$$

Consequently, in order to find the temperature distribution $t_2(x, \tau)$ along the thickness of cloth in the given system, it is sufficient to solve the equation of independent heat transfer;

$$\frac{\partial t_2}{\partial \tau} = a_2 \frac{\partial^2 t_2}{\partial x^2} \qquad (6)$$

for the corresponding constraints.

From the known value of the temperature field $t_2 = f(x, \tau)$ in cloth one can find the optimal time $\tau_{opt}$ of contact between the heat carrier (in this case the cloth) and the dried material (paper), optimal with respect to a rational continuation of the process, and one can also determine the effective values of the heat transfer coefficients at the contact surfaces between cloth and paper.

The solution to Eq. (6) will be arrived at assuming that the thermophysical properties of cloth are constant within the given range of its temperature variation. We have now the following constraints.

Initial condition:

$$t_2(x, 0) = t_2^0. \qquad (7)$$

Third-kind boundary condition at the outer cloth surface

$$\lambda_2 \frac{\partial t_2(0, \tau)}{\partial x} = \alpha \left[t_2(0, \tau) - t_{amb}\right]. \qquad (8)$$

Second-kind boundary condition at the inner cloth surface in contact with paper

$$\lambda_2 \frac{\partial t_2(h, \tau)}{\partial x} = \varphi(\tau). \qquad (9)$$

Function $\varphi(\tau)$ characterizes the quantity of heat expended on heating the paper and vaporizing the moisture:

$$\varphi(\tau) = c_1 g_1 d\bar{t}_1/d\tau + \rho g_1 d\bar{u}_1/d\tau.$$

If the heat required to raise the temperature of paper is negligible, as can be verified by elementary calculations, then

$$\varphi(\tau) = \rho g_1 \frac{d\bar{u}_1}{d\tau}. \qquad (10)$$

It has been established by the authors that the paper between cloth sheets, regardless of the grade of cloth used, becomes dry during the slowing down period and that the curve representing the kinetics of the drying process can be sufficiently well approximated, as in [4, 5], by the equation:

$$\bar{u}_1 - \bar{u}_{1p} = (\bar{u}_{10} - \bar{u}_{1p}) \exp(-2.3\varkappa N\tau). \qquad (11)$$

Then

$$\frac{d\bar{u}_1}{d\tau} = 2.3\varkappa N (\bar{u}_{10} - \bar{u}_{1p}) \exp(-2.3\varkappa N\tau). \qquad (12)$$

Condition (9) will then, with (12) taken into consideration, become

$$\frac{\partial t_2(h, \tau)}{\partial x} = -\frac{g_1\rho}{\lambda_2} 2.3\varkappa N (\bar{u}_{10} - \bar{u}_{1p}) \exp(-2.3\varkappa N\tau) = -km_0 \exp(-m_0\tau), \qquad (13)$$

where

$$k = \frac{g_1\rho}{\lambda_2}(\bar{u}_{10} - \bar{u}_{1p}); \quad m_0 = 2.3\varkappa N.$$

In the way the problem is stated now, questions concerning the mechanism of heat and moisture transfer in the paper are eliminated from consideration. Only questions concerning the transfer in cloth are considered, assuming integral characteristics for the paper.

We note here that a more general version of the problem concerning the contact heating of a moist laminate with evaporation off the free surface of the material has been solved in [6].

In order to find the solution to Eq. (6) with the constraints (7), (8), and (13), we perform a Laplace transformation with respect to variable $\tau$. The solution for the transform becomes

$$\frac{t_2^0}{s} - T(x,\ s) = \left[H(t_2^0 - t_{amb})(s + m_0)\,\mathrm{ch}\sqrt{\frac{s}{a_2}}(h - x) + km_0 s\,\mathrm{ch}\sqrt{\frac{s}{a_2}}x + Hkm_0\sqrt{a_2 s}\,\mathrm{sh}\sqrt{\frac{s}{a_2}}x\right] \Bigg/ \left[s(s + m_0)\left(\sqrt{\frac{s}{a_2}}\,\mathrm{sh}\sqrt{\frac{s}{a_2}}h + H\,\mathrm{ch}\sqrt{\frac{s}{a_2}}h\right)\right]. \quad (14)$$

Using the expansion theorem [3], we find the solution for the original function

$$\frac{t_2^0 - t_2(x,\ \tau)}{t_2^0 - t_{amb}} = 1 - \frac{km_0\cos\sqrt{\frac{m_0}{a_2}}x + Hk\sqrt{m_0 a_2}\sin\sqrt{\frac{m_0}{a_2}}x}{(t_2^0 - t_{cp})\left(\sqrt{\frac{m_0}{a_2}}\sin\sqrt{\frac{m_0}{a_2}}h - H\cos\sqrt{\frac{m_0}{a_2}}h\right)} \times \exp(-m_0\tau) - \sum_{n=1}^{\infty}\frac{A_n}{t_2^0 - t_{cp}}\left\{\left[(t_2^0 - t_{amb})\cos\mu_n - \frac{km_0}{H\left(\frac{m_0 h^2}{a_2\mu_n^2} - 1\right)}\right] \times \cos\mu_n\frac{x}{h} + \left[(t_2^0 - t_{amb})\sin\mu_n - \frac{km_0 h}{\mu_n\left(\frac{m_0 h^2}{a_2\mu_n^2} - 1\right)}\right]\sin\mu_n\frac{x}{h}\right\}\exp\left(-\frac{a_2\mu_n^2}{h^2}\tau\right), \quad (15)$$

where $\mu_n$ are the roots of the equation

$$\mathrm{ctg}\,\mu = \frac{\mu}{\mathrm{Bi}}.$$

Here $\mu = i\sqrt{(s/a_2)}h$; $A_n = 2\sin\mu_n/(\mu_n + \sin\mu_n\cos\mu_n)$, the initial thermal amplitudes, are single-valued functions of the Bi number. The numerical values of the first six amplitudes $A_n$ and roots $\mu_n$ are given in [3].

We rewrite the solution (15) in dimensionless form. The quantity $m_0h^2/a_2$ = Pd is the Predvoditelev number and characterizes the intensity of temperature variations at the inner cloth surface; $a_2\tau/h^2$ = Fo is the Fourier number, and Hh = Bi is the Biot number.

The second term in the numerator on the right-hand side, $k\sqrt{m_0 a_2}/(t_2^0 - t_{amb})$ is the product of $\sqrt{\mathrm{Pd}}$ and the critical number

$$\theta_K = \frac{ka_2}{h(t_2^0 - t_{amb})} = \frac{g_1\rho(\bar{u}_{10} - \bar{u}_{1p})}{c_2\gamma_2 h(t_2^0 - t_{amb})}.$$

The critical number $\theta_K$ is equal to the quantity of heat expended for evaporating the moisture from the paper divided by the quantity of heat which the heat carrier (cloth) can give off while cooling down to ambient temperature $t_{amb}$. This ratio is an analog of the critical number $\theta_M = \rho M_0/[\alpha(T_c - T_0)]$ defined earlier by Lykov to characterize the effectiveness of drying a material by convection [3]. Thus, the critical number $\theta_K$ characterizes the effectiveness of contact drying in the cloth–paper system, in terms of the useful fraction of heat in this process.

The quantity $g_1\rho(\bar{u}_{10} - \bar{u}_{1e})/e_2\gamma_2 h$ is equal to the change in mean cloth temperature $\Delta t = t_2^0 - \bar{t}_2'$ while paper is dried from an initial moisture content $\bar{u}_1 = \bar{u}_{10}$ to $\bar{u}_1 = \bar{u}_{1e}$. In this case

$$\theta_K = \frac{t_2^0 - \bar{t}_2'}{t_2^0 - t_{amb}},$$

i.e., the critical number $\theta_K$ acquires the meaning of a critical parameter.

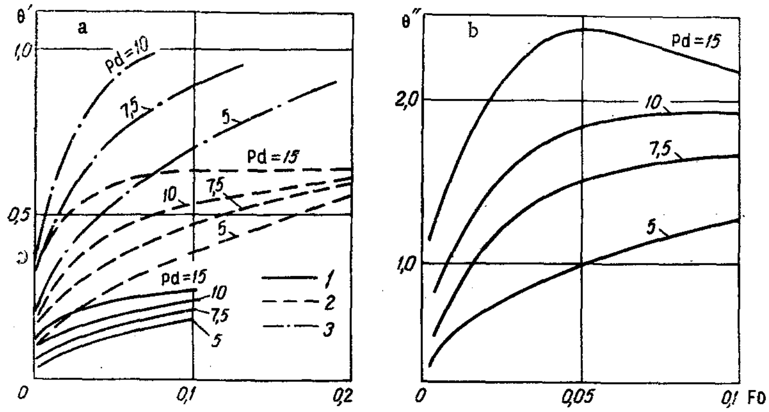


Fig. 1. Curves of referred temperature a) $\theta'$ and b) $\theta''$ for the contact surface of cloth ($x = h$) as functions of the Fourier number (Fo) at various values of critical numbers Pd and $\theta_K$: 1) $\theta_K = 0$; 2) $\theta_K = 0.25$; 3) $\theta_K = 0.5$.

Thus, the solution of (15) in dimensionless form will be

$$\theta = \frac{t_2^0 - t_2(x, \tau)}{t_2^0 - t_{amb}} = 1 - \frac{\left(\mathrm{Pd}\cos\sqrt{\mathrm{Pd}}\,\frac{x}{h} + \mathrm{Bi}\sqrt{\mathrm{Pd}}\sin\sqrt{\mathrm{Pd}}\,\frac{x}{h}\right)\theta_k}{\sqrt{\mathrm{Pd}}\sin\sqrt{\mathrm{Pd}} - \mathrm{Bi}\cos\sqrt{\mathrm{Pd}}}\exp(-\mathrm{Pd\,Fo})$$

$$-\sum_{n=1}^{\infty} A_n\left\{\left[\cos\mu_n - \frac{\theta_k \mathrm{Pd}}{\mathrm{Bi}\left(\frac{\mathrm{Pd}}{\mu_n^2} - 1\right)}\right]\cos\mu_n\frac{x}{h} + \left[\sin\mu_n - \frac{\theta_k \mathrm{Pd}}{\mu_n\left(\frac{\mathrm{Pd}}{\mu_n^2} - 1\right)}\right]\sin\mu_n\frac{x}{h}\right\}\exp(-\mu_n^2\mathrm{Fo}). \quad (16)$$

As a result, the dimensionless cloth temperature $\theta$ is generally a function of the following form

$$\theta = \frac{t_2^0 - t_2(x, \tau)}{t_2^0 - t_{amb}} = f(\mathrm{Bi}, \mathrm{Pd}, \theta_K, \mathrm{Fo}, x/h). \quad (17)$$

From solutions (15) or (16) one can obtain a solution for the case where the outer cloth surface momentarily reaches the ambient temperature $t_{amb}$ (which occurs when cloth is in contact with, say, a surface of metallic cylinders). Then [3] the critical number Bi $\to \infty$, the characteristic numbers $\mu_n = (2n-1)\pi/2$, and the thermal amplitudes

$$A_n = \frac{2\sin\mu_n}{\mu_n} = (-1)^{n+1}\frac{2}{\mu_n} = (-1)^{n+1}\frac{4}{2(n-1)\pi}.$$

After appropriate transformations, Eq. (16) will yield

$$\theta' = \frac{t_2^0 - t_2(x, \tau)}{t_2^0 - t_{amb}} = 1 + \frac{\theta_k\sqrt{\mathrm{Pd}}\sin\sqrt{\mathrm{Pd}}\,\frac{x}{h}}{\cos\sqrt{\mathrm{Pd}}}\exp(-\mathrm{Pd\,Fo})$$

$$-2\sum_{n=1}^{\infty}\left(\frac{1}{\mu_n} - \frac{\theta_k \mathrm{Pd}}{\mu_n^2\sin\mu_n\left(\frac{\mathrm{Pd}}{\mu_n^2} - 1\right)}\right)\sin\mu_n\frac{x}{h}\exp(-\mu_n^2\mathrm{Fo}) \quad (18)$$

or

$$\theta' = f(\mathrm{Pd}, \theta_K, \mathrm{Fo}, x/h).$$

If in Eq. (18) the ambient temperature $t_{amb} = t_2^0$, then the referred variable will be

$$\theta'' = \frac{t_2^0 - t_2(x, \tau)}{t_2^0 - \bar{t}_2'} = \frac{t_2^0 - t_2(x, \tau)}{g_1\rho(\bar{u}_{10} - \bar{u}_{1p})/c_2 g_2} = f(\mathrm{Pd}, \mathrm{Fo}, x/h), \quad (19)$$

where the combination of heterogeneous parameters $g_1\rho(\bar{u}_{10} - \bar{u}_{1e})/c_2 g_2$, as was shown earlier, has the dimension of temperature and may serve here as a specified temperature parameter.

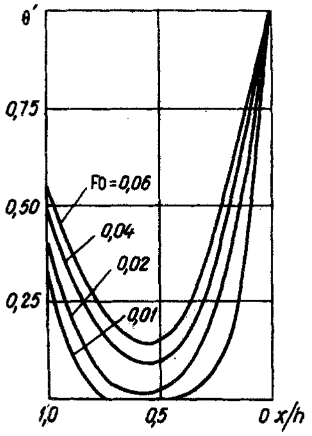


Fig. 2

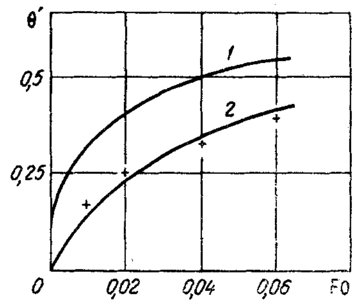


Fig. 3

Fig. 2. Temperature field in the cloth.

Fig. 3. Referred temperature $\theta'$ of cloth as a function of the Fourier number (Fo), (Pd = 12, $\theta_K = 0.25$): 1) x = h; 2) x = 0.9 h. Points indicate test values.

The solution of (18) will be written as

$$\theta'' = \frac{\sqrt{\mathrm{Pd}}\sin\sqrt{\mathrm{Pd}}\,\frac{x}{h}}{\cos\sqrt{\mathrm{Pd}}}\exp(-\mathrm{Pd\,Fo}) + 2\sum_{n=1}^{\infty}\frac{\mathrm{Pd}\sin\mu_n x/h}{\mu_n^2\sin\mu_n\left(\frac{\mathrm{Pd}}{\mu_n^2}-1\right)}\exp(-\mu_n^2\mathrm{Fo}). \tag{20}$$

If $\mathrm{Pd}/\mu_n^2-1 = 0$ in (20), the mean cloth temperature during the entire process remains equal to the initial temperature of the dried material, which can happen when heat is supplied to the outer cloth surface, or when the intensity of the drying process is very low, or when the cloth has a high thermal capacity referred to volume and a high thermal conductivity.

In Fig. 1a, b are shown curves of referred temperature $\theta'$ and $\theta''$ for the cloth surface in contact with paper (x = h), calculated by Eqs. (18) and (20) as functions of the Fo number at various values of the critical numbers Pd and $\theta_K$ within the practical range encountered in paper drying between cloth sheets.

As Fig. 1a, b shows, the curves $\theta'(h, \tau)$ and $\theta''(h, \tau)$ are characterized by a sharp rise at low Fo numbers (sharp drop in temperature at the cloth and paper contact surface) followed by a more even trend at high Fo numbers.

For specific grades of cloth, for examplewoollen drying cloth $g_2 = 3.6$ kg/m$^2$, the variation in referred temperature $\theta'$ and $\theta''$ during the first 5-10 sec of contact time (which corresponds to Fo = 0.01 to 0.02) is greater than during all the next 25-30 sec (Fo = 0.05 to 0.06).

Thus, the analysis shows that already after a few seconds of contact drying the cloth surface temperature drops sharply, i.e., a considerable temperature gradient is established between the center layers of the cloth and those in contact with the paper – as is shown clearly in Fig. 2 taking into account the low thermal conductivity of the material.

In Fig. 2 is shown the profile of referred temperature across the thickness of woollencloth $g_2 = 3.6$ kg/m$^2$ (h = 6.17 mm) for various values of Fo and for Pd = 12 and $\theta_K = 0.25$ constant, which corresponds to the conditions of drying a paper $g_1 = 0.075$ kg/m$^2$ and with $\bar{u}_{10} = 1.95$ kg/kg between cloth sheets at their initial temperature of $t_2^0 = 125$°C. An ambient temperature $t_{amb} = 25$°C is maintained at the outer cloth surfaces throughout the process.

The temperatures in the cloth at a distance of $\delta \approx 0.6$ mm from the contact surface (x = 0.9 h), which have been measured, agree closely with those calculated by Eq. (20) – as is shown in Fig. 3. The calculations indicate that, for an accuracy of $\theta$ to three significant figures after the decimal point, it is sufficient to consider only the first four terms of the summation in (20).

Analysis of the curves points to the fact that the described process of drying is efficient only during the first 5-10 sec of contact between paper and cloth, since further on the intensity of the process decreases considerably, as has been also experimentally confirmed by the authors. In order to ensure high mean drying intensities, it is necessary to maintain the temperature of the cloth surface in contact with the paper at a sufficiently high level throughout the entire drying process. In practice this can be achieved by a stepwise drying of the paper.

On the other hand, the large temperature gradient to be maintained across the cloth layer in contact with the paper is brought about by the relatively low volume heat capacity of cloth grades currently used in practice. In order to carry out the process effectively, therefore, it is advisable to develop and to use for this purpose new grades of cloth with higher volume heat capacities and higher thermal conductivities.

## NOTATION

$a$ is the thermal diffusivity;
c is the specific heat of the material;
$\gamma$ is the density;
$\lambda$ is the thermal conductivity;
$\rho$ is the specific latent heat of evaporation;
h is the cloth thickness;
$\alpha$ is the heat transfer coefficient;
g is the weight of 1 $m^2$ of the material;
N is the maximum drying rate;
$\varkappa$ is the relative drying coefficient;
t is the temperature;
u is the moisture content;
$\tau$ is the time;
$\bar{u}$ is the average moisture content.

### Subscripts

1, 2 denotes to paper and cloth respectively;
0 denotes initial state;
e denotes equilibrium state.

## LITERATURE CITED


1. I. L. Luboshits, G. N. Lebedeva, and L. S. Slobodkin, in: Research on Drying and Thermal Processes [in Russian], Minsk (1968).
2. A. V. Lykov and Yu. A. Mikhailov, Theory of Heat and Mass Transfer [in Russian], Moscow–Leningrad (1963).
3. A. V. Lykov, Theory of Thermal Conductivity [in Russian], Moscow (1967).
4. V. V. Krasnikov and V. A. Danilov, Inzh. Fiz. Zh., 11, No. 4 (1966).
5. A. V. Lykov, Theory of Drying [in Russian], Moscow (1968).
6. M. I. Makovozov, Doct. Dissert., MIKhM (1955).